Online Supplement for Probabilistic vs. Sequence-Based Rendezvous in Channel-Hopping Cognitive Networks

Jelena Mišić, *Senior Member, IEEE* and Vojislav B. Mišić, *Senior Member, IEEE*

This Online Supplement is organized as follows: Section A briefly describes the operation of the transmission tax-based MAC protocol used for probabilistic rendezvous procedure. Section B describes the details of the analytical model for probabilistic rendezvous. Section C describes the details of the analytical model for sequence-based rendezvous.

A Transmission tax-based MAC protocol

Our probabilistic rendezvous mechanism will operate in the context of a recently described MAC protocol [8], [10]. In this protocol, nodes are organized in piconets managed by a coordinator node, similar to in Bluetooth [4]; any node with sufficient computational capability may take up this role. Time is slotted into unit slots and organized in superframes, as shown in Fig. 1. We assume that the superframe contains s_f unit slots, some of which are reserved for administrative purposes such as reporting of sensing results, join/leave and bandwidth reservation requests, beacon and trailer frames. Successive superframes are separated by a guard interval during which all nodes hop to the next channel.

Each node can request time (i.e., bandwidth) for transmitting up to μ data packets. Upon successful transmission, the sender node is obliged to perform sensing for k_p superframes. Sensing nodes independently and randomly select which channels to sense during the data subframe of a superframe, and report the results back to the coordinator in the reporting subframe. The coordinator then compiles and updates a list of idle and busy channels—the channel map—and decides on the channel to be used for the next hop [6]. Due to discrete character of sensing and the delay needed to collect the sensing results, the information in the coordinator's channel map may differ from the actual state [7]; the sensing error may be controlled through judicious choice of k_p [10].

Sensing duty may last for $k_p \geq 1$ superframes, however sensing results are reported in each of these, rather than

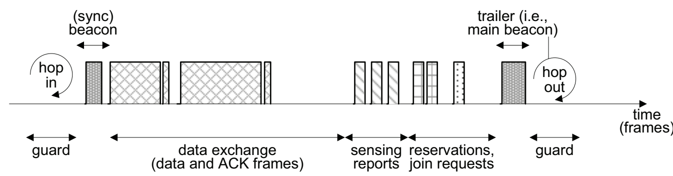


Fig. 1. Superframe and its periods.

at the end of the sensing duty. More frequent reporting ensures synchronization required for channel hopping, reduces the sensing error, and improves throughout by allowing the sensing node to receive data when needed.

Many existing MAC protocols that use superframes require the beacon frame to be sent at the beginning of the superframe; however, the rendezvous protocol is better served by a trailing beacon or trailer. Namely, a node that overhears any valid frame in a superframe with a leading beacon will wait for the reservation sub-frame (which would then be the last one in the superframe) to send a join request. However, such a node could not know which channel to hop to in order to hear the next beacon, and thus would lose synchronization with the piconet. A possible remedy would be to make the coordinator acknowledge a properly received join request packet with an ACK packet indicating the channel to be used for the next hop (and next superframe). However, should the join request packet or the coordinator's ACK packet get lost due to noise and/or interference, the node would still have no idea where to hop next and thus lose synchronization with the piconet it has discovered only a moment ago. A trailing beacon alleviates these risks and allows the node to learn of the next hop channel in time to continue following the piconet, even if it is unable to send a join request in time for the coordinator to hear it.

The trailing beacon includes bandwidth allocation for previously received transmission requests and the next-hop channel. It also includes announcements about join/leave requests granted by the coordinator, as explained in the next subsection.

We note that the superframe structure begins with a small beacon frame sent by the coordinator immediately before the data subframe; this allows nodes to synchronize their transmissions upon the beginning of the superframe, as different

• *J. Mišić and V. B. Mišić are with the Department of Computer Science, Ryerson University, Toronto, ON, Canada M5B 2K3.*
E-mail: jmisic@scs.ryerson.ca, vmisic@ryerson.ca

nodes might require different channel switching time.

B MODELING PROBABILISTIC RENDEZVOUS

Table 1 lists main parameters and variables used to model the probabilistic rendezvous mechanism.

Let us assume that, for any channel, the durations of active and idle times, T_a and T_i, follow mutually independent random probability distributions with the probability density functions (pdf) $t_a(x)$ and $t_i(x)$, respectively. As one cycle time on the channel is comprised of one idle and one active period, the probability density function of the cycle time may be obtained as the convolution of the corresponding pdf's of idle and active periods, i.e., $t(x) = t_i(x) * t_a(x)$. Then, the probability that the channel is busy or idle can be calculated as $p_{on} = \frac{\overline{T_a}}{\overline{T_a} + \overline{T_i}}$ and $p_{off} = 1 - p_{on}$, respectively. The mean cycle time of primary source (hereafter referred to simply as cycle time) will be $\overline{T_{cyc}} = \overline{T_a} + \overline{T_i}$.

Renewal processes used in the analysis. Renewal theory is a major analytical tool used to evaluate the performance of the rendezvous algorithm. A random process which counts the number of some general cycles where cycle durations X_i, are i.i.d. nonnegative random variables is a renewal process [3]. Beginning of a new cycle period is therefore a renewal point at which a new probabilistic replica of the original renewal process starts. If an arbitrary event occurs during general cycle time X_i at a time ω relative to the start of the new cycle (renewal point), the period from onset of the new cycle to the event, $X_{i,-}$, is referred to as deficit (or elapsed) cycle time, and the period from that event to the end of the current cycle $X_{i,+} = X_i - \omega$ as residual (or excess) cycle time.

The following renewal processes may be identified:

1) The process that counts cycles of primary source starting from the onset of idle channel period on some channel σ is a renewal process. Consider the situation where the node visits the target channel when the channel is idle. Let the time of arrival of the node, relative to the renewal point, be denoted as τ. According to renewal theory, $\tau = T_{i,-}$ is referred to as elapsed time and $T_i - \tau = T_{i,+}$ as residual idle time on the channel. Deficit idle channel time has the probability distribution function (PDF) defined as $A(x) = P(\tau < x)$, while its pdf is $a(x) = \frac{dA(x)}{dx}$. Let us also define $P(T_i > x) = T_i^c(x) = \int_{y=x}^{\infty} t_i(y)dy$. Then, the PDF of the deficit channel idle time can be calculated as

$$A(x) = \frac{1}{\overline{T_i}} \int_0^x T_i^c(y)dy \quad (1)$$

and its pdf as

$$a(x) = \frac{d}{dx} A(x) = \frac{T_i^c(x)}{\overline{T_i}} \quad (2)$$

2) The process that counts the number of sensing events on a channel σ is also a renewal process since time periods between two consecutive sensing events follow the same probability distribution derived from the fact that selection of channels to sense is randomly performed by each sensing node, independently of any central authority and the selection of other nodes [7], [9]. For this process, the onset of activity of primary user between two sensing points is a random point in sensing cycle. If we denote duration of sensing period on channel σ as R and the moment of onset of primary user activity relative to previous sensing point as ξ_R, then $\xi_R = R_-$ is elapsed sensing time and $R - \xi_R = R_+$ is residual sensing time. Since sensing periods are synchronized to piconet activity, they are multiples of basic time unit used for MAC design, and the probability distribution of sensing time is discrete, contrary to the distribution of activity times of primary users which are continuous and independent of piconet activities.
In [7], probability distribution of residual sensing time with respect to the start of idle period was calculated via a Probability Generating Function (PGF) of

$$R_+(z) = \sum_{i=0}^{\infty} R_i z^i \quad (3)$$

where mass probabilities R_i depend on the number of nodes in the piconet, traffic load, and scheduling parameter.

3) The process that counts superframes (on any channel) is also a renewal process, although a trivial one. In this case the onset of activity of primary user on channel σ is a random point in a superframe currently running on some channel μ. If we denote duration of superframe as $C = s_f$ and moment of onset of primary user activity relative to start of the superframe as ξ_C, then $\xi_C = C_-$ is elapsed superframe time and $C - \xi_C = C_+$ is residual superframe time. Probability density function of the residual superframe time has the form $c = 1/s_f$ which can be obtained if (2) is applied to constant variable s_f. This result holds for both discrete and continuous versions of the superframe residual time.

As the moments of beginning and end of primary user activity on a given channel occur independently from the arrival of the joining node to the channel, and the piconet arrival to that same channel occurs independently from either of these, interarrival times are independent random variables, and the processes described above are indeed renewal processes.

Delay in piconet access to an idle channel. An idle channel can't be used by the piconet immediately after becoming idle: it must be sensed and recorded as idle in the coordinator's channel map prior to being selected for the next hop. The time between the moment when the channel becomes idle and the moment when the piconet can access it is a random variable and we need to find its distribution. Taking into account the sensing process, channel dynamics, and piconet hopping, we can say that a channel state transition will be sensed within the superframe during which it occurs only if the residual sensing time is shorter than the residual superframe time. If channel state changes during the reporting subframe, sensing of the new channel state will be completed in one of subsequent superframes. Channel transition will be detected in the immediately following superframe if the residual sensing time is longer than the residual superframe time but shorter

TABLE 1
Parameters and variables of the probabilistic rendezvous mechanism.

Piconet parameter	
s_f	superframe size (time units)
Δ	size of reporting, joining field and trailer (time units)
μ	scheduling parameter (packets)
k_p	sensing penalty per packet transmitted (superframes)
Model variable	
N	number of primary channels
T_a, T_i	active and idle time on a channel
p_{off}, p_{on}	probability that a channel is idle (busy)
$P_{\theta 0}$	probability that access to a channel is not possible due to sensing delay
P_{sl}	probability that the trailing edge of channel activity was sensed and reported after l superframes, $l = 0, 1, \dots$
$P_{\theta k}$	probability of access to the channel k superframes ($k = 1, 2, \dots$) after the channel is labeled as available
P_{acc}	probability of successful access to a channel during its idle time
P_{nvis}	probability that a channel will not be accessed during its idle time
a_1	probability of an idle channel incorrectly considered busy
b_1	probability of a busy channel incorrectly considered idle
P_c	probability that residual idle channel time is shorter than superframe duration
P_{col}	probability of collision with primary user transmission
P_{colR}	probability of a collision during rendezvous
P_{ov}	probability of overlap with a piconet superframe
P_{late}	probability of delayed overlap (i.e., the one that extends into the next superframe)
P_{rv}	probability of successful rendezvous
R_{en}	time to successful rendezvous

than the sum of residual superframe time and superframe length; similar reasoning applies for the second, third, ... following superframes.

Probability that trailing edge of channel activity was sensed and reported in the ongoing superframe or l-th superframe afterwards, respectively, can be calculated as

$$\begin{aligned} P_{s0} &= \sum_{k=1}^{s_f} c \sum_{i=0}^{k} R_i \int_{x=k}^{\infty} t_i(x)dx \\ P_{sl} &= \sum_{k=0}^{s_f} c \sum_{i=k+(l-1)s_f}^{k+ls_f} R_i \int_{x=k+ls_f}^{\infty} t_i(x)dx, \quad l > 0 \end{aligned} \tag{4}$$

Then, the probability that access to an idle channel is not possible due to the sensing detection delay at the piconet and the average time after the beginning of idle channel period where access is not possible can be calculated as

$$\begin{aligned} P_{\theta 0} &= \sum_{l=0}^{\infty} P_{sl} \int_{x=0}^{s_f} c \int_{y=0}^{x+ls_f} a(y)dydx \\ \overline{N_{oa}} &= \sum_{l=0}^{\infty} P_{sl} \left(ls_f + c\int_{x=0}^{s_f} xdx \right) = s_f \sum_{l=0}^{\infty} lP_{sl} + \frac{cs_f^2}{2} \end{aligned} \tag{5}$$

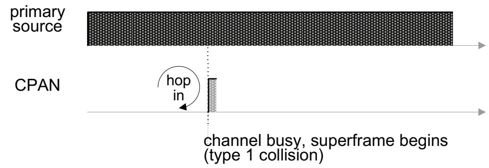
However, when a sufficient number of sensing nodes is available (as is the case for network configurations considered in this work), channel transitions will be detected right away or in the next superframe with high probability.

Probability distribution of the time of piconet access relative to the beginning of channel idle time. After the superframe in which a channel is recorded as idle in the coordinator's channel map, the piconet can access that channel. As the superframe duration s_f is fixed, access can occur at any multiple of s_f slots (and, in fact, more than once) until the channel becomes busy again. Since sensing events are synchronized with the detection time of channel availability, we need to calculate mass probabilities $P_{\theta k}$ of access by the piconet in k-th superframe period ($k = 1, 2, \dots$) after the channel is labeled as available. As channel availability is detected at a random time with respect to beginning of the idle channel period, we can obtain these probabilities as

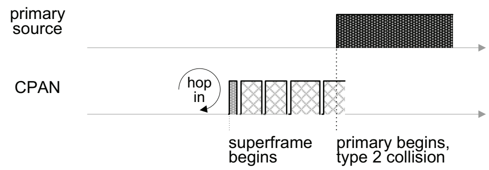
$$\begin{aligned} P_{\theta 1} &= \frac{P_{s0} \int_{x=0}^{s_f} c \int_{y=x}^{x+s_f} a(y)dydx}{1 - P_{\theta 0}} \\ P_{\theta 2} &= \frac{(P_{s0} + P_{s1}) \int_{x=0}^{s_f} c \int_{y=x+s_f}^{x+2s_f} a(y)dydx}{1 - P_{\theta 0}} \\ P_{\theta k} &= \frac{\int_{x=0}^{s_f} c \int_{y=x+(k-1)s_f}^{x+k \cdot s_f} a(y)dydx}{1 - P_{\theta 0}}, \quad k > 2 \end{aligned} \tag{6}$$

Note that access in the first superframe position is possible only if the change of channel status has occurred before the reporting subframe, but this problem may be overcome by considering that the superframe starts at the reporting subframe. In practice, summation of piconet access probabilities is performed up to some limit L which is chosen so that the probability θ_L is smaller than a predefined limit, which was 10^{-4} in our calculations.

Probability of piconet access to the channel. The coordinator's channel map is not perfectly accurate due to the insufficient number of sensing nodes and discrete nature of the sensing process. Let $a_1 \approx 1 - \sum_{l=0}^{\infty} P_{sl}$ denote the probability that the channel map considers a channel where primary activity has ceased to be busy and therefore unusable, and let $b_1 \approx \sum_{i=0}^{\infty} R_i \int_{x=0}^{i} t_a(x)dx$ denote the probability that a busy



(a) Type 1 collision: hopping to a busy channel.



(b) Type 2 collision: channel becomes busy during a superframe.

Fig. 2. Collisions with primary users.

channel is still considered to be idle in the channel map (accurate expressions, as derived in [7], will be used, but we do not present them due to lack of space). Therefore, the probability that the piconet will access a given channel is $p_c = (1-a_1)/(\overline{N_i}(1+b_1))$. Note that, in general, $a_1 \neq b_1$, due to different durations of active and idle channel periods.

Then, the probability that the piconet will access a target channel at least once during its idle time (and not collide with onset of primary activity) can be calculated as

$$P_{acc} = \sum_{l=0}^{\infty} P_{sl} \sum_{k=1}^{L} p_c (1-p_c)^{k-1} \int_{x=0}^{s_f} c\, dx \int_{x+(k+l)s_f}^{\infty} t_i(y) dy - a_1 \tag{7}$$

Note that value a_1 is subtracted since idle channels with obsolete information are considered busy, and therefore will not be chosen for piconet use.

In an analogous fashion, the probability that the piconet will not attempt to access an idle channel is

$$P_{nvis} = \sum_{l=0}^{\infty} P_{sl} \sum_{k=1}^{L} (1-p_c)^{k} \int_{x=0}^{s_f} c\, dx \int_{(k+l)s_f-x}^{(k+l+1)s_f-x} t_i(y) dy + a_1 \tag{8}$$

We also need the probability that the piconet will collide with the primary user on a given channel. Two scenarios may lead to this. A type 1 collision occurs when piconet hops to a busy channel thought to be idle, as shown in Fig.. 2(a); the probability of this scenario is b_1, the probability that the next-hop channel has become active, but the coordinator does not know it since the information in the channel map is obsolete.

A type 2 collision occurs when the primary user becomes active during an ongoing superframe, as shown in Fig. 2(b). The probability of this scenario may be calculated as the probability that residual idle channel time is shorter than superframe duration, i.e.,

$$P_c = \int_{x=0}^{\infty} (A(x+s_f) - A(x)) a(x) dx \tag{9}$$

The total probability of collision between piconet and primary source is

$$P_{col} = b_1 + P_c \tag{10}$$

We note that $P_c + P_{acc} + P_{nvis} = 1$.

B.1 Modeling the delay of the rendezvous process

As explained above, a rendezvous occurs when both the node and the piconet access the same idle channel and the node hears the superframe (or, at least, the superframe trailer) without collision. If the piconet arrives to the channel and begins a new superframe at the time θ, rendezvous will be successful if the time of node arrival and listening τ occurs before the trailer of the said superframe, i.e., if $\tau \leq \theta + s_f - \Delta$. Note that the node will stay at an idle channel for at most T_{wi} slots before switching to another channel; however, this stay may be lengthened if the node hears a valid frame, in which case $\theta \leq \tau + T_{wi} \leq \theta + s_f$.

Probability of direct overlap. Probability that the waiting time of the node will overlap with a superframe, conditioned on the duration of the superframe and the waiting time of the node, is

$$\begin{aligned} P_{ov} = & \sum_{k=1}^{L} P_{\theta k} P(\theta - T_{wi} < \tau < \theta + s_f - \Delta) \\ = & \sum_{k=1}^{L} P_{\theta k} \int_{x=0}^{s_f} c dx \int_{\max(x+(k-1)s_f - T_{wi}, 0)}^{x+ks_f-\Delta} a(u) du \end{aligned} \tag{11}$$

Mean time needed for the node to overlap with piconet on the same channel can be calculated as sum of mean times for cases when $T_{wi} \leq \tau < \theta$, $\tau < \theta < T_{wi}$, and $\theta < \tau < \theta + s_f - \Delta$, respectively:

$$\begin{aligned} \overline{T_{ov}} = & \sum_{k=\min(w+1,L)}^{L} P_{\theta k} \int_{x=0}^{s_f} cdx \int_{x+(k-1)s_f - T_{wi}}^{x+(k-1)s_f} AA \cdot a(u) du \\ & + \sum_{k=1}^{\min(w,L)} P_{\theta k} \int_{x=0}^{s_f} cdx \int_{0}^{x+(k-1)s_f} (u+s_f) a(u) du \\ & + \sum_{k=1}^{L} P_{\theta k} \int_{x=0}^{s_f} cdx \int_{x+(k-1)s_f}^{x+ks_f-\Delta} BB \cdot a(u) du \end{aligned} \tag{12}$$

where $w = T_{wi}/s_f$, $AA = u + s_f - (x + (k-1)s_f - T_{wi})$, and $BB = u - x - (k-1)s_f$.

Probability of delayed overlap. The node may arrive to the channel after the reservation subframe but in time to hear the trailer and obtain the information about the next-hop channel. The node will then follow the piconet to the next superframe and submit a join request in the corresponding reservation subframe. The corresponding probability may be calculated as

$$P_{late} = \sum_{k=1}^{L} P_{\theta k} \int_{x=0}^{s_f} cdx \int_{x+ks_f-\Delta}^{x+ks_f} a(u) du \tag{13}$$

The time spent in waiting is $T_{late} = \Delta + s_f$ slots.

Probability of missed rendezvous. We also need to calculate probability distributions of the total waiting time when the node misses the residence of the piconet on a channel.

1) The node may visit a busy channel, in which case it takes $T_1 = T_{wb}$ time slots to realize that a primary source is active and decide to switch to another channel.

2) The node may visit an idle channel but the piconet does not access that channel during the node waiting time so a rendezvous does not happen. As probability distributions of elapsed and residual idle time on the target channel are identical with the pdf of $a(x)$ [3], we can calculate the waiting time by looking at the end of idle channel time, in which case the mean value of idle waiting time is

$$\overline{T_2} = T_{wi} \int_{x=T_{wi}}^{\infty} a(x)dx + \int_{x=0}^{T_{wi}} (x+T_{wb})a(x)dx \quad (14)$$

3) The node visits an idle channel but its waiting ends before the piconet accesses that channel in the same idle channel period. Probability of this event is

$$P_3^- = \sum_{k=\min(w,L)}^{L} P_{\theta k} \int_{x=0}^{s_f} cdx \int_0^{\max(0,x+(k-1)s_f-T_{wi})} a(y)dy \quad (15)$$

and mean waiting time is

$$\overline{T_3^-} = T_{wi} P_3^- \quad (16)$$

4) The node visits an idle channel after the piconet has left and rendezvous does not occur. Let variable y model the arrival time to the channel. The probability of this scenario is

$$P_3^+ = \sum_{k=1}^{L} P_{\theta k} \int_{x=0}^{s_f} cdx \int_{y=x+(k)s_f}^{\infty} a(y)dy \quad (17)$$

As before, mean waiting time is calculated by exploiting the symmetry between residual and elapsed idle time, whilst using the end of idle time as the reference point. Considering the remaining channel idle time after the piconet leaves the channel, the following cases are possible:

- The remaining channel idle time is shorter than the maximum waiting time of the node, $T_{wi} = ws_f$.
- The remaining channel idle time may be longer than the maximum waiting time of the node, and the time interval between the arrival of the node and the end of idle period on the channel is shorter than T_{wi}.
- The remaining channel idle time may be longer than the maximum waiting time of the node, but the time interval between the arrival of the node and the end of idle period on the channel is longer than T_{wi}.

The three cases outlined above correspond to three components of the mean waiting time:

$$\begin{aligned}\overline{T_3^+} = & \sum_{k=1}^{\min(w,L)} P_{\theta k} \int_{x=0}^{s_f} cdx \int_0^{y=x+(k-1)s_f} ya(y)dy \\ & + \sum_{k=\min(w+1,L)}^{L} P_{\theta k} \int_0^{y=ws_f} ya(y)dy \\ & + \sum_{k=\min(w+1,L)}^{L} P_{\theta k} \int_{x=0}^{s_f} cdx \int_{y=ws_f}^{x+(k-1)s_f} T_{wi}a(y)dy \end{aligned} \quad (18)$$

Waiting time when a rendezvous is destroyed by collision. A pending rendezvous can be destroyed by the onset of primary user activity on the channel. (Note that the presence of the new node makes this event different from a collision of only piconet with the primary user activity.) Probability that a collision occurs during rendezvous and mean time spent in waiting for a rendezvous that will ultimately fail are found by considering the superframe during which the channel changes state and calculating the time relative to the end of idle channel period:

$$P_{colR} = \int_{x=0}^{s_f} \left(\int_{y=0}^{x+T_{wi}} a(y)dy \right) a(x)dx \quad (19)$$

$$\overline{T_4} = \int_{x=0}^{s_f} a(x)dx \int_{y=0}^{x+T_{wi}} ya(y)dy \quad (20)$$

Distribution of unsuccessful waiting time on an idle channel. Since we can find Laplace-Stieltjes Transform (LST) of time intervals $T_j, j = 1 \,..\, 4$, as $T_j^*(s) = \int_{x=0}^{\infty} e^{-xs} f_j(x)dx$, we can derive the LST for the waiting time on the channel when rendezvous was missed as

$$\begin{aligned} M^*(s) = & p_{on}e^{-sT_{wb}} + p_{off}P_{nvis}T_2^*(s) + p_{off}P_{col}T_4^*(s) \\ & + p_{off}P_{acc}(P_{ov} + P_{late} + P_3^+ T_3^{+*}(s) + P_3^- T_3^{-*}(s)) \end{aligned} \quad (21)$$

The average waiting time, then, is $\overline{M} = -M'^*(0)$.

The probability of successful rendezvous is conditioned by the need for the piconet and the node to access the same idle channel and the constraint on the overlap between their respective residence times:

$$P_{rv} = p_{off}(1-a_1)(1-P_{colR})(P_{ov} + P_{late}) \quad (22)$$

With all the components in place, we can describe the probability distribution of the time needed for a successful rendezvous with the LST of

$$\begin{aligned} R_{en}^*(s) = & P_{rv} \frac{T_{ov}^*(s)P_{ov} + e^{-s(s_f+\Delta)}P_{late}}{P_{ov} + P_{late}} \\ & \cdot \sum_{k=0}^{\infty} ((1-P_{rv})M^*(s))^k \\ = & \frac{T_{ov}^*(s)P_{ov} + e^{-s(s_f+\Delta)}P_{late}}{1-(1-P_{rv})M^*(s)} \cdot \frac{P_{rv}}{P_{ov} + P_{late}} \end{aligned} \quad (23)$$

and its mean and standard deviation are

$$\begin{aligned} \overline{R_{en}} & \quad -\frac{d}{ds} R_{en}^*(s)\big|_{s=0} \\ var(R_{en}) & = \frac{d^2}{ds^2} R_{en}^*(s)\Big|_{s=0} - \overline{TTR_M}^2 \end{aligned} \quad (24)$$

Coefficient of variation of this probability distribution is therefore $CV = \sqrt{var(R_{en})}/\overline{R_{en}}$. Note that all higher moments of this probability distribution can be derived from (23); in fact, the entire pdf can be obtained using inverse Laplace transform [5].

C Modeling sequence-based rendezvous

Table 2 lists main parameters and variables used to model the sequence-based rendezvous mechanism.

TABLE 2
Parameters and variables of the probabilistic rendezvous mechanism.

Model variable	
$s_l = N(N+1)$	length of the sequence
$P_{lag,i}$	probability of lag of i slots between the sequences of initiator and follower nodes
$P_{one,i}$	probability that the sequence is broken by primary user activity on the rendezvous channel
P_d	probability that a rendezvous will be destroyed

While sequence-based algorithms promise a bounded maximum TTR in the absence of primary user activity, the situation radically changes when cognitive nodes attempt to co-exist with primary users. In this case, three important events can occur on an idle channel, as shown in Fig. 3.

1) The initiator node begins its rendezvous sequence from channel σ at the time κ, measured relative to the start of idle time on the said channel.
2) The follower node begins its rendezvous sequence from channel ϵ at the time ψ after the beginning of the idle period on that channel. Since the beginning of the idle periods on channels σ and ϵ are independent events, time of arrival of follower is also a random event with respect to the start of idle period at channel σ. We will denote time of arrival of follower relative to start of σ's idle time as ψ'.
3) Given the lag of $0 \,..\, s_l - 1$ r-slots between initiator and follower and the (possible) difference between their individual sequences, rendezvous will occur at some channel μ. However, rendezvous may be interrupted by the onset of primary channel activity at time ξ relative to the beginning of the idle period on channel μ. Since the idle periods on channels σ and μ are independent, the beginning of activity at channel μ will be a random event with respect to the start of idle time at channel σ. Let ξ' denote the beginning of primary user activity at rendezvous channel μ relative to start of idle time at channel σ.

The conclusion is simple: in the presence of random primary user activity, time to rendezvous becomes a random variable, and it must be calculated using probabilistic tools, similar to the case of probabilistic rendezvous.

In the analysis that follows, we will use the orthogonal sequence algorithm from [2], [11] as a representative sequence-based algorithm. Our objective is to demonstrate how the upper bound for rendezvous time (which is $s_l = N(N+1)$ rendezvous slots for a network with N channels) changes in the presence of random primary user activity on these N channels; in particular, we show that the upper bound becomes a random variable itself, and hence can't have a fixed value as is the case in the absence of primary user activity. This observation holds for other sequence-based algorithms as well.

C.1 Probability of lagging between sequences

Let us consider the follower node which arrives to the channel ϵ and begins its sequence; at that time, the initiator can be in any of the s_l r-slots of its sequence. The probability that the follower node comes to idle channel ϵ before the completion of i-th r-slot, but after the completion of the $i-1$-th one, conditioned upon the event that the initiator sequence begins at time $\kappa = x$, is $P(\kappa = x)P(x + (i-1)s_r < \psi' < x + is_r)$. By unconditioning for $0 \le x \le \infty$, we obtain the probability of lagging as

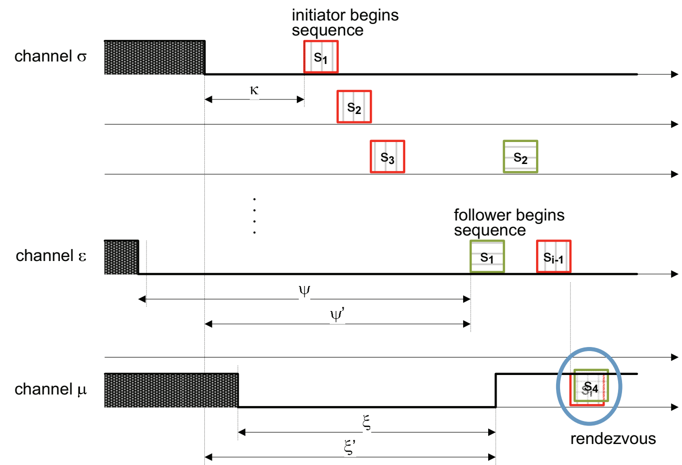


Fig. 3. Timing on initiator and follower node starting channels (σ and ϵ, respectively) and rendezvous channel μ.

$$P'_{lag,i} = \int_{x=0}^{\infty} P(\kappa = x)P(x + (i-1)s_r \le \psi' \le x + is_r)dx = \int_{x=0}^{\infty} \left(A(x + is_r) - A(x + (i-1)s_r)\right) a(x)dx \qquad (25)$$

where $i \in (1 \,..\, s_l)$. To obtain the probability distribution of the lag, we need to scale those probabilities with their sum:

$$P_{lag,i} = P'_{lag,i} \Big/ \sum_{i=1}^{sl} P'_{lag,i} \qquad (26)$$

C.2 Probability of destroyed rendezvous

We need to find the probability that the rendezvous channel μ becomes active before or at rendezvous point. Since the position of rendezvous depends on lag (denoted with i r-slots) and follower's sequence, we will assume that the probability of rendezvous occurring in k-th r-slot in the follower sequence is $Pr_{i,k}$ where $\sum_{k=1}^{s_l} Pr_{i,k} = 1$. The probability of primary activity on channel μ between the start of the initiator sequence and k-th slot of the follower sequence (which lags i r-slots can be calculated as

$$\begin{aligned} P_{one,i,k} &= \int_{x=0}^{\infty} P(\kappa = x) Pr_{i,k} P(x \le \xi' \le x + (i+k)s_r) dx \\ &= Pr_{i,k} \int_{x=0}^{\infty} \frac{1}{s_l} \left(A(x+(i+k)s_r) - A(x)\right) a(x) dx \end{aligned} \tag{27}$$

Assuming that the initiator sequence has started when all N channels were idle, (27) holds for any channel from the sequence. Probability that the initiator/follower sequence is broken by primary user activity on the rendezvous channel is

$$\begin{aligned} P_{one,i} &= \sum_{k=1}^{s_l} P_{one,i,k} \\ &= \sum_{k=1}^{s_l} Pr_{i,k} \int_{x=0}^{\infty} \left(A(x+(i+k)s_r) - A(x)\right) a(x) dx \end{aligned} \tag{28}$$

By unconditioning $P_{one,i}$ on the lagging value i, we find the probability that a rendezvous will be destroyed as

$$P_d = \sum_{i=1}^{s_l} P_{lag,i} P_{one,i} \tag{29}$$

C.3 TTR in the presence of primary activity

To characterize the probability distribution of TTR, let us denote the maximum TTR without primary user activity as TTR_m. Then, the PGF of a broken follower sequence can be modeled using uniform distribution as $T_r(z) = \sum_{i=1}^{TTR_m} \frac{1}{TTR_m} z^i$. As the time between two consecutive rendezvous attempts is subject to different policies depending on the actual protocol, we will simply model it with a PGF $I(z)$. Then, the PGF for the maximum TTR becomes

$$\begin{aligned} TTR_M(z) &= (1-P_d) z^{TTR_m} \\ &\quad + P_d T_r(z) I(z) (1-P_d) z^{TTR_m} \\ &\quad \dots \\ &\quad + (P_d T_r(z) I(z))^i (1-P_d) z^{TTR_m} \\ &= \sum_{i=0}^{\infty} (P_d T_r(z) I(z))^i (1-P_d) z^{TTR_m} \\ &= \frac{(1-P_d) z^{TTR_m}}{1 - P_d T_r(z) I(z)} \end{aligned} \tag{30}$$

Finally, the mean and variance of the maximum TTR become

$$\begin{aligned} \overline{TTR_M} &= \left. \frac{d}{dz} TTR_M(z) \right|_{z=1} \\ var(TTR_M) &= \left. \frac{d^2}{dz^2} TTR_M(z) \right|_{z=1} + \overline{TTR_M} - \overline{TTR_M}^2 \end{aligned} \tag{31}$$

C.4 Sequences with multiple rendezvous points

Our analytical model can easily be extended to sequences with multiple rendezvous points on different channels, such as the one proposed in [1], provided that sequence break is modeled on each channel. To this end, let us assume an optimal sequence with the length $O(N^2)$, with a total of N rendezvous points on N different channels $\mu_1, \mu_2, \dots \mu_N$, with indices assigned to channels according to the order in which they occur in the rendezvous sequence. As before, we use the start of idle period on channel σ as reference point, with the follower sequence lagging by i r-slots behind the initiator, and primary activity on channels μ_j occurring at moments ξ'_j. We also need to partition the whole sequence into N sections so that each section contains a single rendezvous opportunity, assuming that a rendezvous can occur uniformly over l slots in the section of the sequence with a single rendezvous. Then, the probability that a rendezvous will be destroyed by primary activity may be calculated using the approach outlined above. We can then use this result to obtain the probability that the rendezvous will succeed in m attempts and, consequently, the maximum value of TTR.

C.5 Additional remarks

So far we have been looking at primary user activity only on the channel on which the rendezvous will occur. However, we can take a more conservative approach and require that all of the channels visited during the initiator and follower sequences must be idle from the beginning of the initiator sequence up to and including the rendezvous r-slot. If this is not the case, the busy channel may be excluded from the sequence [2], [11] and the sequences will need to be recalculated (since the number of channels has changed) and restarted. In general, different nodes will decide that a channel is not feasible, and subsequently recalculate and restart their sequences, at different times. The drawback of this approach is that the sequences can easily exhaust the available channels, even though some of the busy channels may actually become idle again. The proposed restart of sequences will, thus, prolong the time to achieve rendezvous. However, the new rendezvous point may itself be destroyed by primary user activity, and the analysis presented above remains valid.

REFERENCES

[1] K. Bian, J.-M. Park, and R. Chen. Control channel establishment in cognitive radio networks using channel hopping. *IEEE J. on Selected Areas in Communications – Wireless Series*, 29(4):689–703, 2011.

[2] L. DaSilva and I. Guerreiro. Sequence-based rendezvous for dynamic spectrum access. In *3rd IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN 2008)*, pages 1–7, Oct. 2008.

[3] D. P. Heyman and M. J. Sobel. *Stochastic Models in Operations Research, Volume I: Stochastic Processes and Operating Characteristics*. McGraw-Hill, New York, 1982.

[4] IEEE. Wireless medium access control (MAC) and physical layer (PHY) specifications for wireless personal area networks (WPAN). IEEE Std 802.15.1 (Revision of IEEE Std 802.15.1-2002), IEEE, New York, NY, 2005.

[5] L. J. Kleinrock. *Queuing Systems*, volume I: Theory. John Wiley and Sons, New York, 1972.

[6] B. Li, P. Yang, J. Wang, L. Shen, Y. Xu, Y. Liu, and X.-Y. Li. Optimal action point for dynamic spectrum utilization under Rayleigh fading. *Ad Hoc and Sensor Wireless Networks*, 17(1–2):1–32, 2013.

[7] J. Mišić and V. B. Mišić. Performance of cooperative sensing at the MAC level: Error minimization through differential sensing. *IEEE Transactions on Vehicular Technology*, 58(5):2457–2470, June 2009.

[8] J. Mišić and V. B. Mišić. Simple and efficient MAC for cognitive wireless personal area networks. In *Proc. Global Telecommunications Conference GLOBECOM'09*, Honolulu, HI, Nov. 2009.

[9] J. Mišić. Cooperative sensing at the MAC level in simple cognitive personal area networks. *IEEE J. on Selected Areas in Communications – Wireless Series*, 30(9):1711–1718, 2012.

[10] V. B. Mišić and J. Mišić. Cognitive MAC protocol with transmission tax: Dynamically adjusting sensing and data performance. In *Proc. Global Telecommunications Conference GLOBECOM'10*, Miami, FL, Dec. 2010.

[11] N. C. Theis, R. W. Thomas, and L. A. DaSilva. Rendezvous for cognitive radios. *IEEE Transactions on Mobile Computing*, 10(2):216–227, Feb. 2011.