Deriving implications

We may derive an implication $a \to b$ by first assuming a and then reasoning through logical forms and equivalences that b follows.

$$\begin{array}{rrc}
\text{Assume } a & & \text{(Assumption)} \\
& \vdots & \text{logical} \\
& \vdots & \text{deduction} \\
& b & \text{(End Assumption)} \\
\therefore a \to b & &
\end{array}$$

Knights and Knaves

Explanation

On a certain island there are two types of people, **Knights** and **Knaves**.

Knights always tell the truth, so any statement made by them can be assumed to be true.

Knaves **always** lie, so any statement made by them can be assumed to be false.

The original Knights and Knaves problem is the following:

You are visiting the island of knights and knaves and wish to visit a famous ruin. On your way you reach a fork in the road and are unsure of which direction to take. By the side of the road there are two natives, the first one says: 'Watch out he's a knave', to which the other replies Oh no, he's the knave.'

What single question can you ask (of either) to find the way?

Formal rendering of Knights and Knaves

Here we give some explanaition of Knights and Knaves problems.

1. All the knights and knaves problems assume that each person encountered is either a knight or a knave, but not both. More formally, if we decide to represent the statement P is a knight by p, then the statement P is a knave is represented by $\sim p$

2. All knights and knaves problems assume that knights always tell the truth and knaves always lie. This means that each utterance any of the character makes is formalized into 2 statements:

   If the character is a knight then the utterance is true *and*
   If the character is a knave, then the utterance is false.

Formally this means that if P's statement is q:

$$\begin{array}{rcll} p & \rightarrow & q & \textit{and} \\ \sim p & \rightarrow & \sim q & (\equiv q \rightarrow p). \end{array}$$

3. In knights and knaves we usually assume one of the characters is a knight or a knave and derive a contradiction, the rule of contradiction $\left(\dfrac{p \rightarrow c}{\therefore \sim p}\right)$ then tells use that the opposite is true. Using p for P is a knight and q for P's statement we reason as below

$$\begin{array}{rrl} \text{Assume } p & & \text{(Assumption)} \\ & q & \text{(so P's statement is true)} \\ & \vdots & \\ & c & \text{(End Assumption)} \\ p \rightarrow c & & \text{(Proof of implication)} \\ \sim p & & \text{(Contradiction)} \end{array}$$

or

$$\begin{array}{rrl} \text{Assume } \sim p & & \text{(Assumption)} \\ & \sim q & \text{(so P's statement is false)} \\ & \vdots & \\ & c & \text{(End Assumption)} \\ \sim p \rightarrow c & & \text{(Proof of implication)} \\ p & & \text{(Contradiction)} \end{array}$$