# Knights and Knaves

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## 1 Deriving implications

We may derive an implication $a \to b$ by first assuming $a$ and then reasoning through logical forms and equivalences that $b$ follows.

$$\begin{array}{rc}
\text{Assume } a & \text{(Assumption)} \\
\vdots & \text{logical} \\
\vdots & \text{deduction} \\
b & \text{(End Assumption)} \\
\therefore a \to b &
\end{array}$$

## 2 Knights and Knaves

### 2.1 Explanation

On a certain island there are two types of people, **Knights** and **Knaves**.
Knights always tell the truth, so any statement made by them can be assumed to be true.
Knaves **always** lie, so any statement made by them can be assumed to be false.
The original Knights and Knaves problem is the following:
You are visiting the island of knights and knaves and wish to visit a famous ruin. On your way you reach a fork in the road and are unsure of which direction to take. By the side of the road there are two natives, the first one says: 'Watch out he's a knave', to which the other replies Oh no, he's the knave.'
What single question can you ask (of either) to find the way?

### 2.2 Formal rendering of Knights and Knaves

Here we give some explanaition of Knights and Knaves problems.

1. All the knights and knaves problems assume that each person encountered is either a knight or a knave, but not both. More formally, if we decide to represent the statement P is a knight by $p$, then the statement P is a knave is represented by $\sim p$

2. All knights and knaves problems assume that knights always tell the truth and knaves always lie. This means that each utterance any of the character makes is formalized into 2 statements:

   If the character is a knight then the utterance is true *and*
   If the character is a knave, then the utterance is false.

Formally this means that if P's statement is $q$:

$$\begin{array}{rcll} p & \to & q & \textit{and} \\ \sim p & \to & \sim q & (\equiv q \to p). \end{array}$$

3. In knights and knaves we usually assume one of the characters is a knight or a knave and derive a contradiction, the rule of contradiction $\left(\dfrac{p \to c}{\therefore \sim p}\right)$ then tells use that the opposite is true. Using $p$ for P is a knight and $q$ for P's statement we reason as below

$$\begin{array}{rrl} \text{Assume } p & & \text{(Assumption)} \\ & q & \text{(so P's statement is true)} \\ & \vdots & \\ & c & \text{(End Assumption)} \\ p \to c & & \text{(Proof of implication)} \\ \sim p & & \text{(Contradiction)} \end{array}$$

or

$$\begin{array}{rrl} \text{Assume } \sim p & & \text{(Assumption)} \\ & \sim q & \text{(so P's statement is false)} \\ & \vdots & \\ & c & \text{(End Assumption)} \\ \sim p \to c & & \text{(Proof of implication)} \\ p & & \text{(Contradiction)} \end{array}$$