RYERSON UNIVERSITY

DEPARTMENT
OF
MATHEMATICS

MTH 110	**Final Exam**	**December 6, 2008**
Total marks: 100		**Time allowed: 3 Hours.**

NAME (Print): ______________________ **STUDENT #:** ______________

SIGNATURE: ______________________________

Circle your Lab Section:

Section 1 Thursday 9 VIC 101	Section 2 Monday 11 VIC 201	Section 3 Tuesday 9 VIC 301
Section 4 Tuesday 11 VIC 202	Section 5 Friday 9 VIC 310	Section 6 Thursday 11 VIC 106

Instructions:

- Verify that your paper contains 8 questions on 8 pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cell-phones, pagers and ipods must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.

6 Mk

1. Use the truth table below to determine whether the equivalence below is true. Be sure to show how you are determining equivalence.

$$\sim((q \wedge p) \Rightarrow (r \vee q)) \equiv c$$

Where c represents a contradiction.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

8 Mk

2. Use standard set identities (Theorem 5.2.2 etc.) to show that

$$(A - B)^c \cap (A \cap B)^c = A^c.$$

Be sure to justify each step.

3. You are traveling on the island of knights and knaves and come across two natives, A and B. They make the following statements:

   A: Both of us are knights!
   B: If A is a knight then I'm a knave!

   (Recall all inhabitants of the island are either knights or knaves, knights always tell the truth and knaves aways lie.)

   Let a and b denote the statements 'A is a knight' and 'B is a knight' respectively.

   2 Mk (a) Write out the statements of A and B in symbolic form using only the standard logical symbols and a and b.

   A:

   B:

   Warning: The next two questions depend on this one, but will be marked independently. Check your answer carefully before proceeding.

   9 Mk (b) Show that A is a knave. (Your answer should be in the form of a logical symbolic deduction with each step justified.)

   9 Mk (c) What is B? Justify your answer, you may assume $\sim a$ from part (b). (Your answer should be in the form of a logical symbolic deduction with each step justified.)

4. Prove that if the sum of two integers is even then either they are both even, or they are both odd.

15 Mk

Use the predicates $E(x)$ and $O(x)$ to represent 'x is even' and 'x is odd' respectively.

You may also assume the *Parity Theorem*, that an integer is even if and only if it is not odd, i.e. $\forall x \in \mathbb{Z}, E(x) \Leftrightarrow \sim O(x)$

Be sure to justify each step and lay out your proof correctly.
Note that marks will be given for the translation into symbolic form.

5. Let P be the statement

$$\forall\ a, b, c \in \mathbb{Z},\ (a|b \ \wedge \ a \nmid c) \Rightarrow a \nmid (b + c).$$

3 Mk

(a) Give the converse of P in symbolic form and give a counterexample to show that the converse of P is false.

3 Mk

(b) Give the negation of P in symbolic form.

12 Mk

(c) Prove P by contradiction. Be sure to clearly state any assumptions you are making, justify each step and lay out your proof correctly.

6. Define the equivalence relation D on $\mathbb{Z}^2$ by

$$\forall (a,b),(c,d) \in \mathbb{Z}^2,\ (a,b)D(c,d) \Leftrightarrow a \equiv c \bmod 2 \text{ and } b \equiv d \bmod 3.$$

4 Mk

(a) List four elements of the equivalence class [(5,3)].

6 Mk

(b) How many equivalence classes of D are there in total? List a representative element of each of them.

15 Mk

7. Define the order relation R on $S = \{x \in \mathbb{Z}^+ \mid x \leq 7\}$ by

$$\forall a, b \in S,\ aRb \Leftrightarrow \text{Either } a = b, \text{ or } a \bmod 3 < b \bmod 3.$$

(a) Is D a total or a partial order? Justify your answer.

(b) Either find a greatest element or give two non-comparable maximal elements.

(c) Either find a least element or give two non-comparable minimal elements.

(d) Give a chain of length 2.

(e) Draw the Hasse diagram of R.

(f) Show that R is antisymmetric.

8. Let $f : \mathbb{R} \longrightarrow \mathbb{Z}$ be the floor function $f(x) = \lfloor x \rfloor$ and $g : \mathbb{Z} \longrightarrow \mathbb{Q}$ be given by $g(x) = x + \frac{1}{2}$.

8 Mk

(a) Show that f is onto.

(b) Show that f is not 1-1.

(c) Show that g is 1-1.

(d) Is $f \circ g$ 1-1? Prove or give a counterexample.

(e) Is $f \circ g$ onto? Prove or give a counterexample.