RYERSON UNIVERSITY

DEPARTMENT OF MATHEMATICS

MTH 110 | **Midterm Test I** | **October 30, 2009**

Total marks: 60 | **Time allowed: 110 Minutes**

NAME (Print): _____ **STUDENT #:** _____

SIGNATURE: _____

Circle your Lab Section:

Section 1 Wednesday 11 VIC 304	Section 2 Wednesday 5 VIC 201	Section 3 Wednesday 1 VIC 201
Section 4 Wednesday 4 VIC 201	Section 5 Wednesday 12 VIC 301	Section 6 Wednesday 3 VIC 201

Instructions:

- Verify that your paper contains 8 questions on 7 pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cell-phones, and ipods must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.

9 Mk

1. Use the standard set identities (Theorem 5.2.2 etc.) to show that the following set identity holds

$$(A - (B \cap C)) - B^c = (A \cap B) - C$$

6 Mk

2. Use the truth table below to determine if the argument form below is valid or invalid. Be sure to clearly label the premises and conclusions and to show how you are reaching your conclusions.

$$\begin{array}{l} p \Rightarrow (r \wedge q) \\ \sim r \vee q \\ \hline \therefore \sim p \end{array}$$

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

7 Mk

3. Find the bitwise **AND, OR** and **XOR** of 0xCA with 173. Write your answers in the space provided, in the format given there. Be sure to show all your work, marks will be given for the translation to and from binary.

AND	**OR**	**XOR**
Decimal	Hexadecimal	Hexadecimal

4. (a) Find the boolean expression coresponding to the I/O table below. Do not simplify your answer, i.e. leave it in disjunctive normal form.

2 Mk

P	Q	R	Output
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

6 Mk

(b) Draw the corresponding digital circuit.

5. Let $A_n = \{x \in \mathbb{N} \mid x < n\}$.
   (Recall $\mathbb{N} = \{0, 1, 2, \ldots\}$.)

7 Mk

(a) Write out the elements of A_0, A_1 and A_2.

(b) Find $A_2 \cap \{2\}$.

(c) Find $A_2 - A_0$.

(d) Find $A_1 \times A_2$.

(e) Find the power set of A_2, $\mathcal{P}(A_2)$.

(f) Find $A_n \cup \{n\}$.

(g) Is $\{A_n, \{n\}\}$ a partition of A_{n+1} for $n > 0$? Explain your answer.

6. Consider the following statement S:

$$\forall x, y \in \mathbb{Z},\ (x > 0 \wedge y > 0) \Rightarrow x + y > 0$$

In answering this question be sure to quote any rules your use.
(Recall that prenex form has all quantifiers at the beginning and the rest of the statement in Conjunctive Normal Form (CNF).)

8 Mk

(a) Translate S into English.
Note that your answer must not contain any variables or quantifiers and marks will be given for the succinctness of your answer.

(b) Give the **converse** of S in symbolic form. Give your final answer in prenex form, be sure to show your working. You may assume that the atoms are of the form $x > 0$.

(c) Show that the converse is false.

(d) Give the **negation** of S in symbolic form. Give your final answer in prenex form, be sure to show your working. You may assume that the atoms are of the form $x > 0$.

(e) Translate the negation of S into English.
Note that your answer must not contain any variables or quantifiers and marks will be given for the succinctness of your answer.

6 Mk

7. Give each of the following statements about a world W in Tilomino syntax. Your answer should be in a form that would be acceptable to the Tilomino program.

   (a) All small triangles are above a square.

   (b) For a triangle to be below a square it is sufficient that it be small.

9 Mk

8. Suppose that we are given the set of premises:

$$
\begin{array}{ll}
1) & \sim p \Rightarrow q \\
2) & p \Rightarrow (r \wedge s) \\
3) & \sim s \\
4) & \sim q \vee r
\end{array}
$$

Use standard argument forms (Modus Ponens and so forth) and logical equivalences to find the truth values for each of p, q, r and s. Put your final answers (T or F for each) in the box below.

Show your working and be sure to justify each step, stating which of the standard valid forms or logical equivalences you have used.

p	
q	
r	
s	