

RYERSON UNIVERSITY

DEPARTMENT
OF
MATHEMATICS

MTH 210

Midterm Test I

March 3, 2006

Total marks: 50

Time allowed: 110 Minutes

NAME (Print): _____ STUDENT #: _____

Circle your Lab Section:

Monday 11
BUS 300
021

Wednesday 10
EPH 112
011

Wednesday 10
BUS 210
031

Instructions:

- Verify that your paper contains 6 questions on 6 pages.
 - You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
 - No other aids allowed. Electronic devices such as calculators, cellphones, pagers and ipods must be turned off and kept inaccessible during the test.
 - Please keep your Ryerson photo ID card displayed on your desk during the test.
 - In every question show all your work. The correct answer alone may be worth nothing.
 - Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
 - Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.
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1. Prove by induction that

$$\sum_{i=2}^n i(i-1) = \frac{n(n-1)(n+1)}{3}$$

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2. Consider the sequence $a_0 = 1$, $a_n = 3a_{n-1} + 2$, $n > 0$.

- (a) Calculate a_1 , a_2 , a_3 , a_4 . Keep your intermediate answers as you will need them for the next part of the question.

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Mk

- (b) Use iteration to find a formula for the sequence. Simplify your answer as much as possible. In particular, your final answer should not contain sums. Quote any formulas or rules that you use.

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Mk

3. Find a regular expression for the following languages over $\{0, 1\}$

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(a) All string which **do not** begin with a 1.

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Mk

(b) All strings in which every odd position is a 1.

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Mk

4. Find all trees with five vertices up to isomorphism.

5. For each of the languages, L , over $\Sigma = \{a, b\}$ defined by the given regular expression:

- i. Determine if $\epsilon \in L$;
- ii. give two nonempty strings in L
- iii. give two nonempty strings which are in Σ^* but not in L .
- iv. Give a *short* description of L in English.

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Mk

(a) $(a^* \vee b^*)$

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(b) $(ab \vee ba \vee bb)(a \vee b)^*$

6. For each of the following either explain why the specified graph cannot exist (quote any theorems you use), or draw a graph with the given property.

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(a) A graph with degree sequence 3, 3, 3.

(b) A graph with degree sequence 3, 3, 4.

(c) A simple bipartite graph on 6 vertices containing an Eulerian circuit and a Hamiltonian circuit.

(d) A simple graph with 3 connected components on 5 vertices.

(e) A binary tree of height 3 with 9 leaves.

(f) A binary tree with 6 vertices and 3 leaves.